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Simulation-based analysis of the retry queueing system M/G/1 with impatient consumers, collisions, and an unreliable server

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Abstract

In this study, we analyze an M/G/1 type retrial queueing system with impatient customers, collisions, and an unreliable server. Our study is innovative in that we do a sensitivity analysis using various customer service time distributions on important performance metrics, such as the likelihood of desertion, the average waiting time of a randomly selected, successfully serviced, impatient client, etc. If a consumer does not receive the proper service after a predetermined, random waiting period, they have the option to leave the system in orbit; these customers are referred to as impatient customers. Requests can still reach the system in the event of a server failure; however, they will be sent directly to the orbit. It is intended for the operation, repair, impatience, retry, and service times to be independent of one another. A number of graphical representations show how the studied distributions are compared, as well as the intriguing results produced by our own simulation tool. The obtained findings are contrasted with those of [2] in order to demonstrate the benefits of doing simulations in specific situations and to see how the system characteristics alter when other service time distributions are used.

Keywords: Retrial queue, Impatient customers, Collisions, Unreliableserver, Sensitivity analysis.

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1. Introduction

Even for systems that are already in place, it is imperative to create and use new techniques and concepts for communication system design due to the increasing number of users, devices, and networks. Big data transmission results from an increasing number of businesses across all industries expanding their services through cloud networking and a greater number of devices. Because these studies can ease the difficulty of changing or developing systems, it is thus essential to create mathematical and simulation models of contemporary telecommunications networks. When consumers confront crowded service units in real life, they frequently decide to try to get help after a random amount of time has passed in the system. These clients may be described using retrial queues and are situated in an orbit virtual waiting room as opposed to a queue. Retrial queueing systems are frequently used to mimic new issues in large-scale telecommunications networks, such call centers and telephone switching systems. Many papers dealt with these types of systems which can be viewed in the following works like in [4, 9, 19].

A number of researchers have already examined models including consumers' irritation in lines, such as the reneging and balking processes. The majority of current findings about systems possessing the impatient attribute are available, for instance, in [7, 8, 16]. A conflict may arise when, in some circumstances, a second message appears in the channel during the transmission of the first, rendering both unable to decode. This may occur as a result of the restricted number of communication channels or, occasionally, of haphazard attempts that result in the loss of transmission and the subsequent need for retransmission. When this happens, the requests go into orbit, and new attempts to contact the service facility will be made after an arbitrary amount of waiting time. It is necessary to look into and establish effective protocols for averting disagreements and the ensuing communication delays. Of course, there are papers that have studied retrial queues with collisions see for example [11],[12],[13],[14],[15].

It is expected that the system's components are always accessible while looking through the material that is currently available. In reality, this is highly improbable, therefore scientists must investigate the dependability of retrial queueing systems since server malfunctions and maintenance significantly impact the system's characteristics and performance metrics. Typical issues in real-world systems include power outages, human error, and wireless communication packet transmission failures or disruptions during transfer. Sadly, these issues can occur at any time. These systems with an unreliable server were analyzed in several papers, for example in [3],[6],[10],[18],[20].

A retrial M/M/1 type queueing system with a Poisson flow of arrivals, impatient consumers, collisions, and an unreliable service device is presented in the study [2]. In that, the stationary distribution of the number of consumers in the orbit is defined using an asymptotic analysis approach. We study the identical model as in [2], but our simulation program package collects the data. This method makes it feasible to compute performance metrics for which it is nearly difficult to find precise formulae using numerical or asymptotic analysis. There are many software programs available that can evaluate and describe complex systems if all of the random variables are exponentially distributed. However, using simulation has a huge benefit in that any other distribution can be included into the code in addition to the exponential one. Our study is unusual in that we have incorporated other service time distributions into the previously created models and performed a sensitivity analysis to determine whether the reported oddities hold true for this model or how this alteration affects the performance measurements. We choose stochastic simulation to do this because it makes it possible to compute the required measures and get analytical results—which, in this context, are difficult to obtain, if at all possible. We offer graphical findings demonstrating intriguing occurrences with the aid of this application.

2. System model

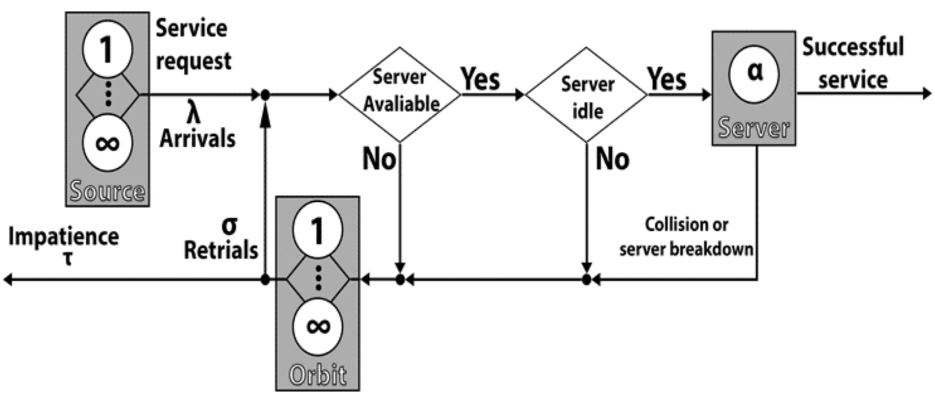

Figure 1: The considered model.

We investigate the M/G/1 queueing system depicted in Figure 1, which has collisions, impatient clients, and an unstable server. The Poisson process with a rate of λ characterizes the system arrival process. When a new customer arrives, the service unit is immediately put into an idle state. The service is distributed using random variables that have the same mean value and variance but differing parameters, such as exponential, gamma, Pareto, lognormal, hypo-exponentially, and hyper-exponentially distributed. If not, it is sent in the direction of the orbit. It is presumed that the requests' retry times have an exponential distribution at a rate of σ. When a consumer arrives at a busy server, the collision causes both requests to enter the orbit. Since it is assumed that the server is unstable, it malfunctions periodically according to an exponential distribution with parameter $\gamma 0$ for idle times and parameter $\gamma 1$ for busy times. During that time, fresh requests were generated goes on, but they are all sent into space. It is sent for repair right away after a breakdown, and the recovery procedure is also an exponential random variable with a rate of $\gamma 2$. Every consumer has the ability to be "impatient," which means they might leave the system sooner than expected after waiting an arbitrary amount of time in the orbit .The impatient time distribution is parametrized by τ and has an exponential distribution. It is assumed in this faulty model that requests are promptly sent into orbit following an interruption or breakdown. All services, including those that are disrupted, operate independently of one another.

3. Simulation Results

The findings of our simulation program are obtained using a statistical tool that was created by Andrea Francini in 1994 [5]. This tool allows one to use the batch means approach to obtain a quantitative estimate of the mean and variance values of the specified variables. Each batch has n observations, and the usable run is split up into a certain number of batches. For the estimator to function well, the batches must be sufficiently lengthy and roughly independent. It is among the most often used confidence interval methods for determining a process's steady-state mean. More in-depth details regarding this technique are available in the ensuing papers in [1]. There is a 99.9% confidence level in the simulations. 0.00001 is the relative half-width of the confidence interval that must be reached in order to terminate the simulation.

4. First Scenario

In order to compare the performance measurements with one another, four distinct service time distributions are included in the sensitivity analysis realization. Every time, the parameters are chosen so that the variance and mean are the same. In order to do that, we employed a fitting procedure that must be followed; the complete procedure is described in full in [17], which also includes a description of each distribution that was used. To examine the impact of the different distributions, two scenarios are created. While Table 1 displays the values of the other parameters, Table 2 displays the parameters that were selected for the service time distribution. In the first, the hyper-exponential, gamma, Pareto, and lognormal distributions are applied when the squared coefficient of variation is larger than one. Due to page restrictions, results related to the second scenario—where the squared coefficient of variation is less than one—were also reviewed; however, they will only be published in the expanded edition of the work.

σ	γ0	γ1	γ2	τ
0.01; 0.001	0.1	0.2	1	0.02; 0.002

Table 1: Numerical values of model parameters

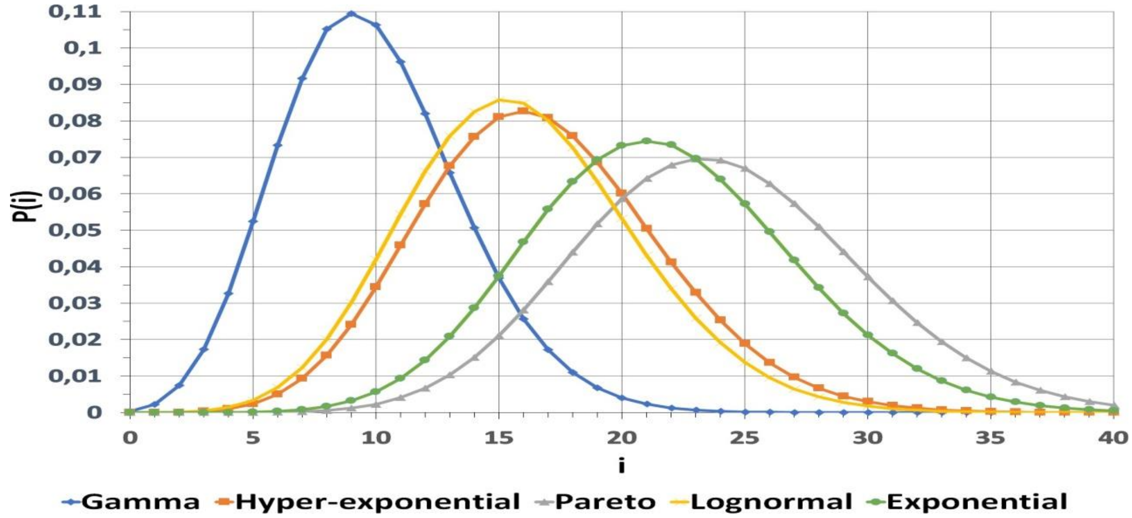

Figure 2: Distribution of the number of customers in the orbit using various distributions $\sigma = 0.01$, $\tau = 0.02$, $\lambda = 0.7$

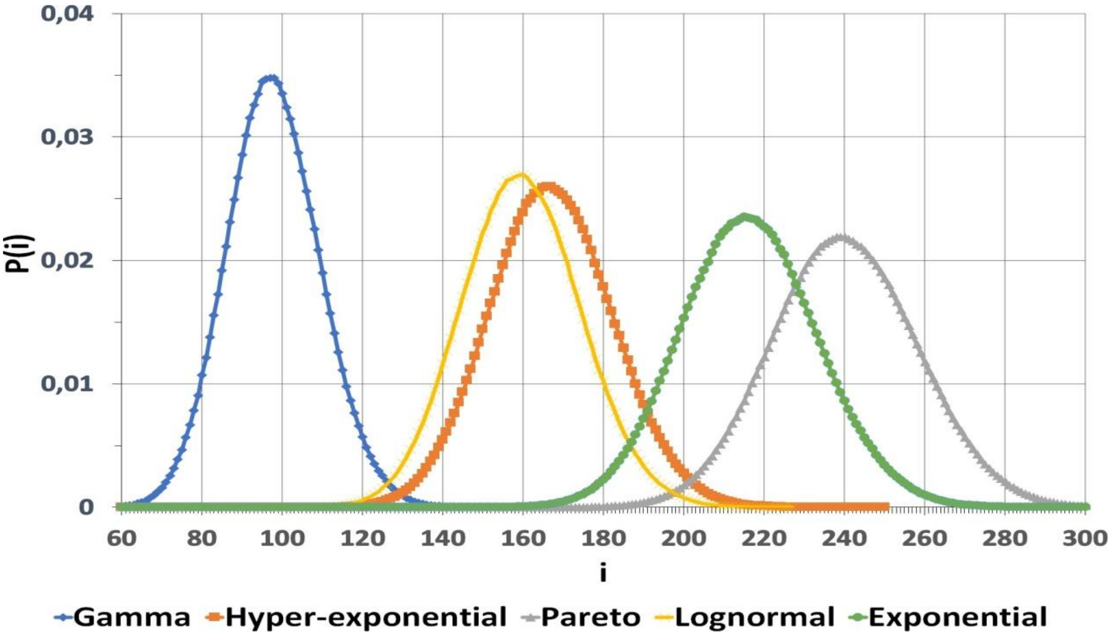

Figure 3. Distribution of the number of customers in the orbit using various distributions $\sigma = 0.001$, $\tau = 0.002$, $\lambda = 0.7$.

Distribution	**Gamma**	**Hyper-exponential**	**Pareto**	**Lognormal**
Parameters	$\alpha = 0.0816$ $\beta = 0.0816$	$p = 0.4607$ $\lambda_1 = 0.9214$ $\lambda_2 = 1.0786$	$\alpha = 2.040$ $k = 0.5098$	$m = -1.292$ $\sigma = 1.6075$
Mean	1			
Variance	12.25			

Squared coefficient of variation	12.25

Table 2: Parameters of service time of incoming customers

When the distribution of the arriving customers' service time is different, Figures 2 and 3 provide a comparison of the steady-state distribution of the number of consumers in the orbit. It shows the likelihood that there will be a certain number of consumers (i) residing in the orbit (P (i)). Upon closer inspection, the curves exhibit a normal distribution independent of the parameter settings that were applied. Along with the other applicable distributions, the pictures also depict the situation of an exponential distribution with the same mean. There is a notable variation in the average number of customers in the orbit; at the gamma distribution, clients spend the least amount of time waiting, while at the Pareto distribution, customers spend the most time waiting.

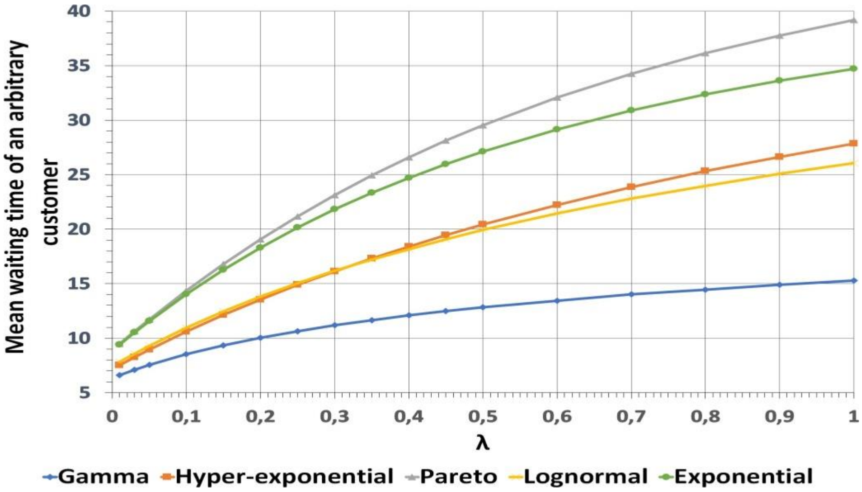

Figure 4: Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions, $\sigma = 0.01$, $\tau = 0.02$.

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Figures 4 and 5 show the mean waiting time of an arbitrary client as a function of the intensity of incoming customer arrivals. Large disparities arise between the applied distributions even when the variance and mean are the same. An arbitrary customer's mean waiting time grows in tandem with the increase in arrival intensity.

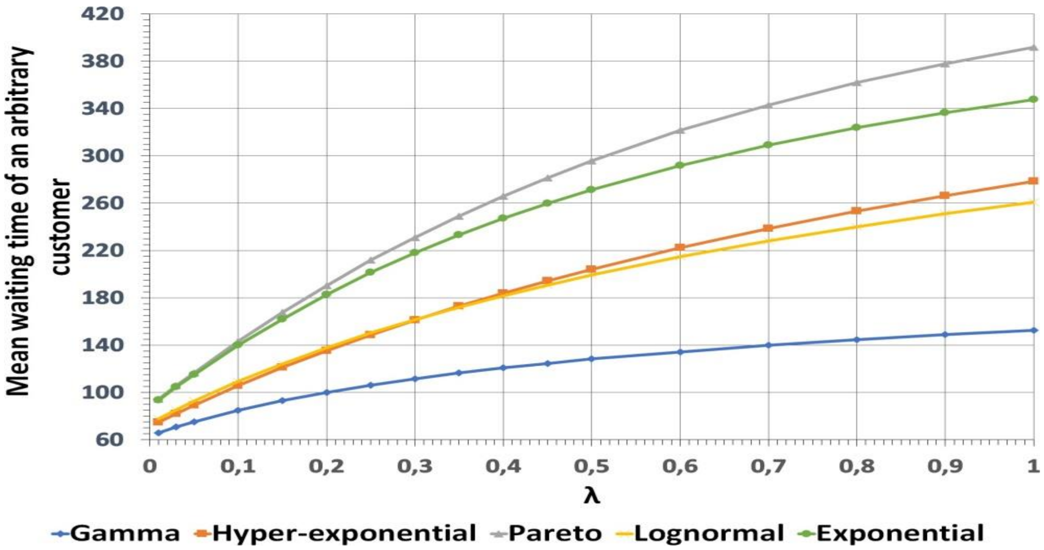

Figure 5: Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions, $\sigma = 0.001$, $\tau = 0.002$

When we adjust the impatient time and retry values, the same effect is seen. The mean waiting time is reduced when the gamma distribution is used instead of the others, particularly when compared to the gamma and Pareto distributions.

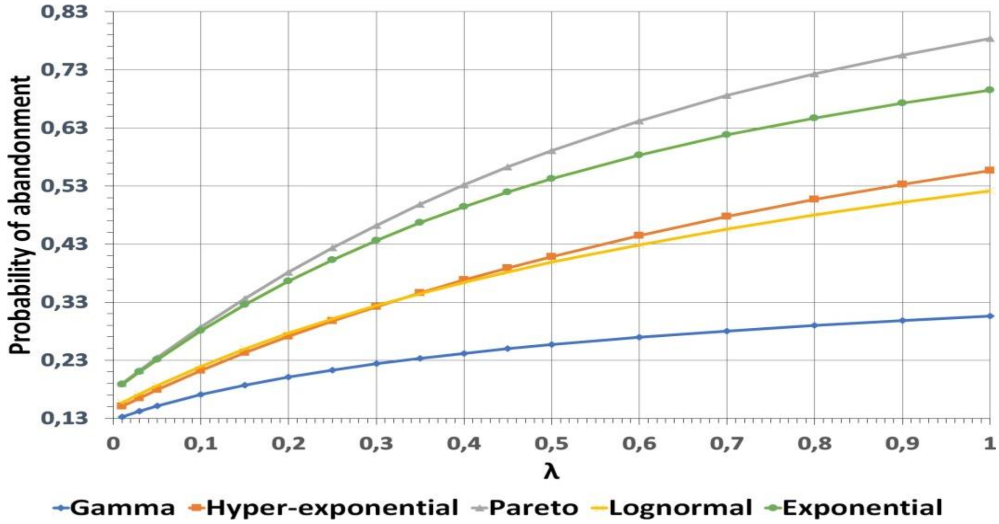

Figure 6: Comparison of probability of abandonment, $\sigma = 0.01$, $\tau = 0.02$.

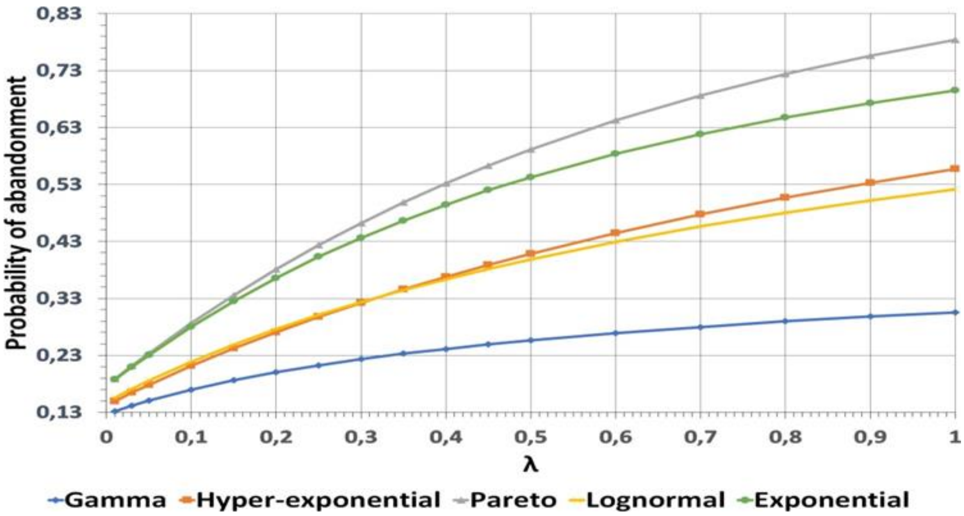

Figure 7: Comparison of probability of abandonment, $\sigma = 0.001$, $\tau = 0.002$.

Along with growing arrival intensity, Figures 6 and 7 show how the chance of a client abandoning you develops. This metric indicates the likelihood that a random user would disconnect from the system at any point during the orbit, indicating that the request did not receive the necessary level of service (patient users). This performance measure's value rises with increasing λ, as is the case for all utilized distributions, however the differences between them are very substantial. The gamma distribution exhibits a far lower tendency of early departure than the Pareto and exponential distributions. Examining Figures 6 and 7 in more detail reveals that the resulting values of this measure are nearly similar since the connection between σ and τ is still the same.

5. **Second Scenario**

We adjusted the settings of incoming customers' service time after analyzing the findings and trends from the preceding section to observe how this new parameter setting affected the performance metrics. Since the squared coefficient of variation in this case is less than one, a hypo-exponential distribution was utilized in place of a hyper-exponential one. We review the identical numbers as in the previous case, but this time we apply the new service time restrictions, which are shown in 3. Every other characteristic stayed the same (refer to Table 1).

Distribution	**Gamma**	**Hypoexponential**	**Pareto**	**Lognormal**
Parameters	$\alpha = 1.6$ $\beta = 1.6$	$\mu 1 = 4$ $\mu 2 = 1.3333$	$\alpha = 2.6125$ $k = 0.6172$	$m = -0.2428$ $\sigma = 0.6968$
Mean	1			
Variance	0.625			
Squared coefficient of variation	0.625			

Table 3: Parameters of service time of incoming customers

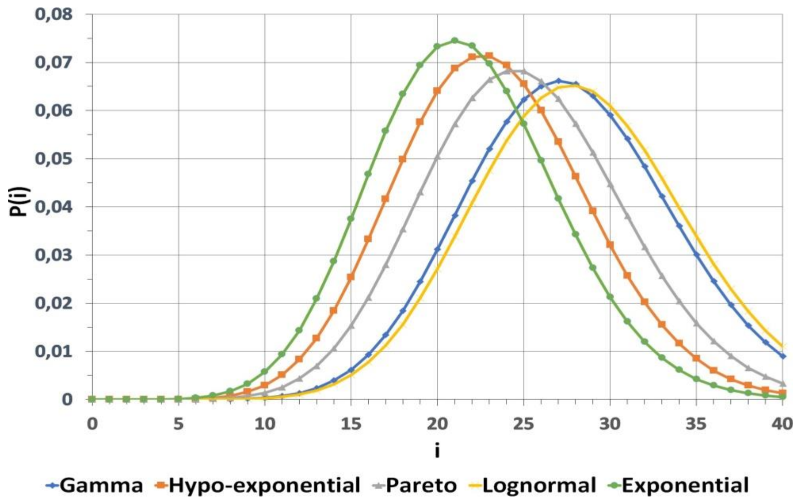

Figure 8: Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.01$, $\tau = 0.02$, $\lambda = 0.7$.

The steady-state distribution of the number of customers in the orbit utilizing different service time distributions is shown in Figures 8 and 9. Even while the difference is still very considerable in Figure 9, the resulting curves are significantly closer to each other with this parameter value, and the curves' shapes reflect the normal distribution. In contrast to Figures 2 and 3, the mean number of customers is larger in the cases of gamma and lognormal distributions.

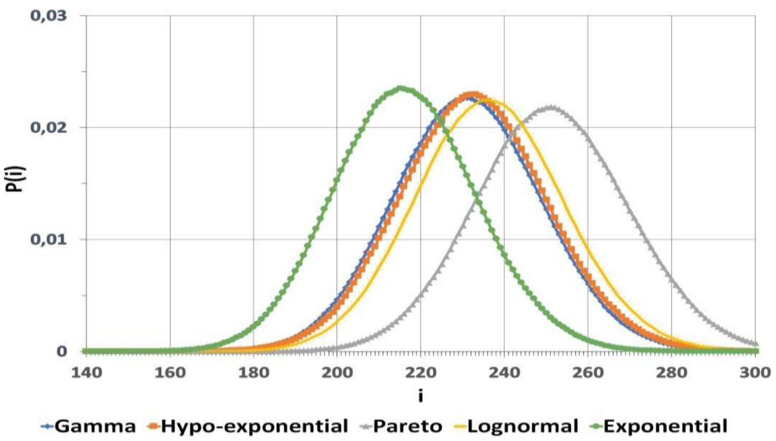

Figure 9. Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.001$, $\tau = 0.002$, $\lambda = 0.7$.

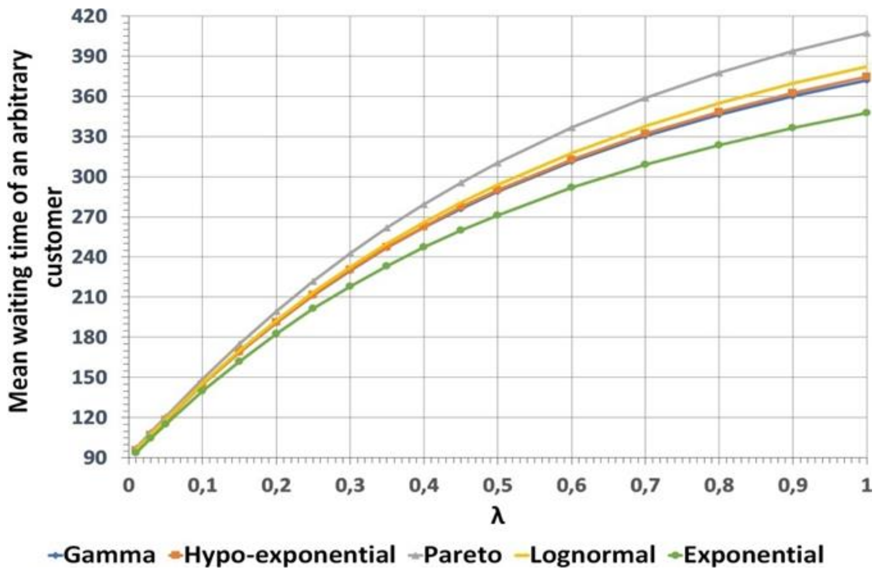

Figure 10: Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions, $\sigma = 0.01$, $\tau = 0.02$.

The mean waiting time of a random client is the subject of the next two figures (Figures 10 and 11). After analyzing the data, it can be concluded that, while the numbers in the Pareto distribution are somewhat greater, there are actually very little changes. Other than that, they nearly exactly match, and both figures show the same ten-dency. As arrival intensity increases, so does the mean waiting time. The obtained findings clearly show that, when comparing the applicable distributions collated in this scenario with the previous one, the features of the system are different when employing these parameters of service time. Lastly, in order to compare all of the possibilities that were looked into The probability of abandoning as a function of arrival intensity is shown in Figure 12. It seems sense how this metric evolves after looking at the two earlier numbers. In addition to increased arrival intensity, the realized values demonstrate how closely the applied distributions match one another and the likelihood that any one consumer would exit the system from its orbit. The data displayed are nearly exact replicas of Figure 12's findings for $\sigma = 0.001$ and $\tau = 0.002$, as discussed in the preceding section.

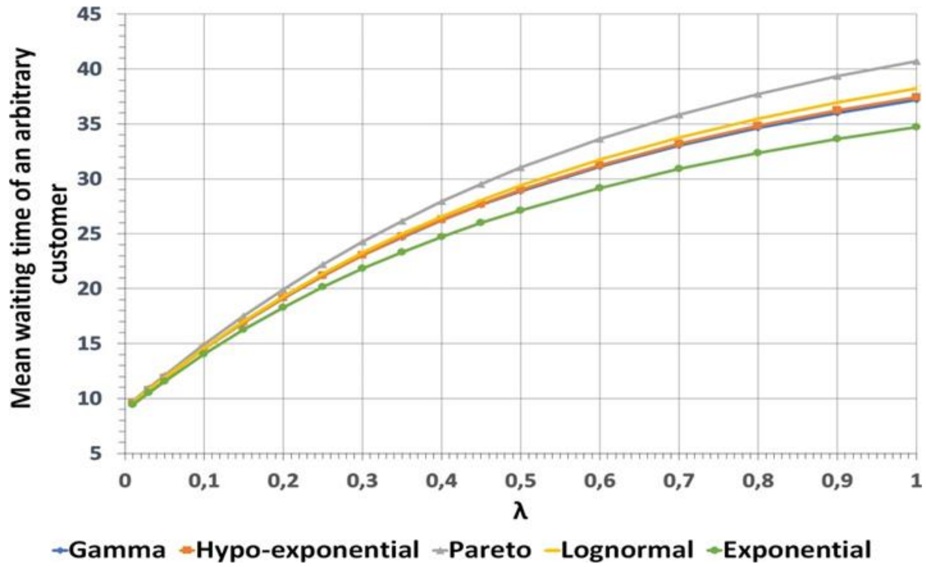

Figure 11: Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions,σ = 0.001, τ = 0.002.

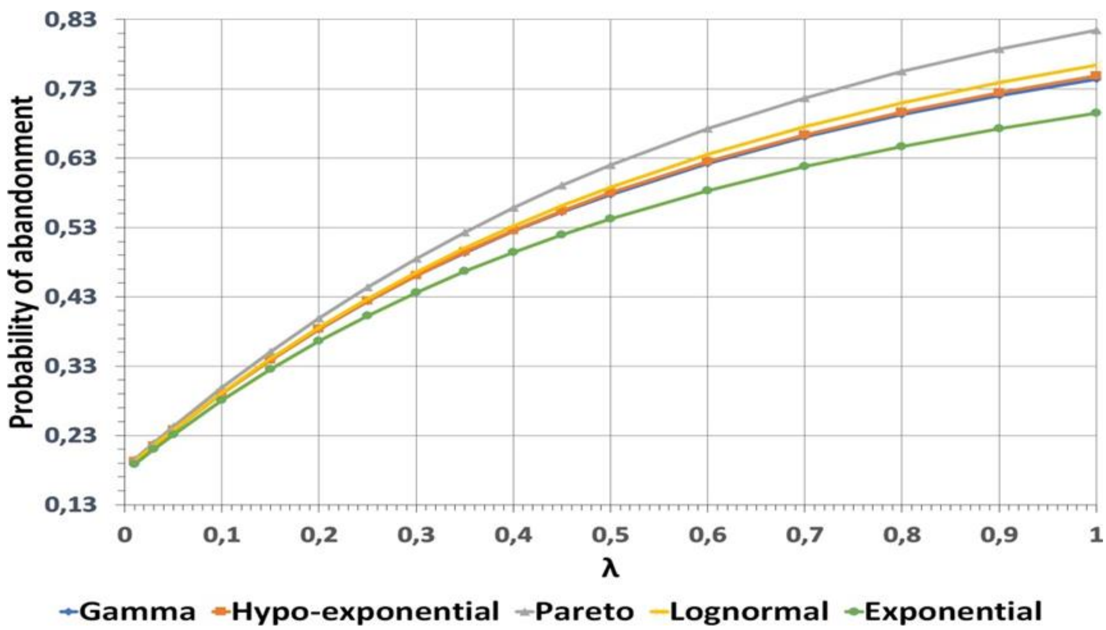

Figure 12: Comparison of probability of abandonment, σ = 0.01, τ = 0.02.

6. Conclusions

In a retrial queueing system of type M/G/1 with an unreliable server and impatient consumers in the orbit, we explored the evolution of performance metrics such as the mean number of customers in the orbit or the mean waiting time of an arbitrary client. After the simulation was run, the results showed that, for every applied distribution, the number of customers in the orbit matched the normal distribution. Additionally, when the squared coefficient of variation is greater than one, it is shown how the various distributions impact the performance measures even when the mean value and variance are equal. Results clearly showed a modest influence on the performance measures in comparison to the first scenario in the other situation, where the squared coefficient of variation is less than one. We would enhance this sensitivity study in the future by adding more distributions and adding functionality to the system, such as two-way communication or alternative modes of operation in the event of a server failure.

7. References

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